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STRESSES AROUND CIRCULAR, SQUARE AND OPTIMIZED HOLES IN CIRCULAR CYLINDRICAL SHELLS UNDER TORSION

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Abstract—This paper presents optimized hole shapes in circular cylindrical shells under torsion for a range of values of β, the curvature parameter. The hole shapes are optimized considering only the predominantly large membrane stresses around the holes, with the use of two-dimensional photoelastic method and a special polariscope arrangement and the removal of material from low stress regions around the hole in a systematic way. The membrane stress distributions around circular, square (with one of its diagonals along the axis of the shell) and optimized holes are obtained. The membrane stress concentration factor for the optimized holes decreased by as much as 23% and 33% when compared to circular and square holes respectively, while the area covered by the optimized hole increased by as much as 41% when compared to the circular hole. Coefficient of efficiency of about 0.90 is achieved for optimized holes. The geometries of the optimized holes are presented in a form suitable for use by designers. Studies on square holes show that, while the s.c.f. for square hole is less than the s.c.f. for the corresponding circular hole for $\beta < 0.75$, the opposite is true for $\beta > 0.75$.

INTRODUCTION

Hole shapes which yield minimum stress concentration factor (s.c.f.) and exhibit an optimum distribution of stresses in cylindrical tubes and shells are of considerable interest to designers of weight conscious and fatigue prone engineering structures such as those used in aerospace vehicles. A classical example of such an optimum hole shape is that of a small elliptical hole ($b/a = \frac{1}{2}$) with its major axis along the hoop direction in the case of a circular cylindrical shallow shell under internal pressure [1]. Obviously, development of methods for the determination of such optimum hole shapes for other loading conditions has considerable significance. It has been shown recently [2, 3] that the two-dimensional photoelastic method, which has been widely used in hole shape optimization in plates [4–6], can be successfully adapted for such an optimization in shells also. Results in the form of optimized hole shapes, stresses around them and their s.c.f. were presented for circular cylindrical shells under axial tension for different values of the curvature parameter β. The purpose of the present paper is to make use of the same basic idea for hole shape optimization in circular cylindrical shells under torsion and present optimized hole shapes and their corresponding s.c.f.s for a range of values of β. Results are also generated for square holes with one of their diagonals parallel to the axis of the shell. The present values are compared with those available in literature for circular and square holes. These include the preliminary results presented in [3] for small holes in shells under torsion.

BASIC APPROACH

A study of the stress distribution around holes in shells shows that the stresses have two components, viz. membrane and bending. For circular holes in circular cylindrical shells under torsion, it is seen from Peterson [1] that membrane stresses are larger than the bending stresses with the bending stress component reaching a maximum value of about 42% of the total stress for $\beta = 1.2$ (Fig. 1). Since simultaneous consideration of both the membrane and the bending stresses for optimization purposes would be very involved, it is proposed to optimize the hole shape by taking into account only the predominantly large membrane stresses.

Defining an optimized hole as one in which uniform membrane stresses are present respectively in the tensile and compressive segments of the hole boundary, it is found

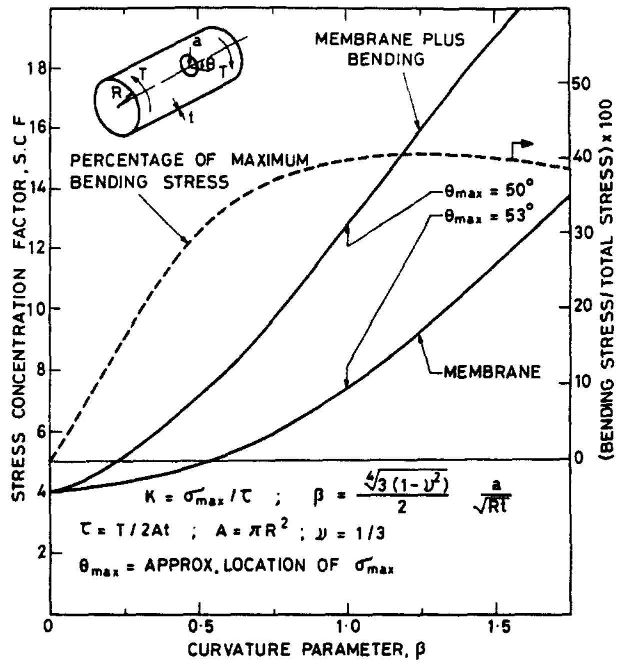


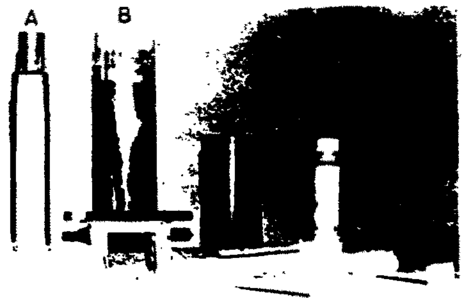
Fig. 1. Membrane and bending s.c.f.s for a circular hole in a circular cylindrical shell under torsion.

that the method of two-dimensional photoelasticity can be used with a special purpose polariscope to achieve optimized hole shapes using shell models made of birefringent materials [2, 3]. In the present investigation, optimum hole shapes are determined by considering membrane stresses only.

MODEL PREPARATION

Two integrally cast circular cylindrical epoxy shells, A and B, were made with shell A having Dobeckot resin 505/c and Hardner 758 in the ratio 100: 10 and shell B having Dobeckot resin 504/c, Hardner 750 and Dibutyl Phthalate in the ratio 100: 10:2 by weight respectively. A cavity for casting the shell was prepared using a thin polythene sheet with its outer surface coated with plaster of Paris for the outer mould with a central circular inner core made of a true chrome-plated metal cylinder coated with a silicone releasing agent. After casting and curing, with the epoxy shell in the metal cylindrical core in position, the polythene sheet reinforced with the plaster of Paris mould was removed carefully and the outer surface of the epoxy shell was turned on a lathe to the required thickness. The present casting method with the use of polythene sheet makes the removal of plaster of Paris outer mould very easy and avoids damage to the epoxy shell, unlike the one quoted in Refs. [2] and [3] where a wax coating was given to the plaster of Paris shell. The end portions of the shell were reinforced up to a distance of 25 mm with the same material to facilitate attachment to the end fixtures. After complete curing, the epoxy shell was removed from the metal core using a hydraulic press. The surface was in need of a thin coating of oil to get good transparency. Typically the shell internal diameter was measured to be 162.5 ± 0.05 mm. The different stages of making shells for the experimental purpose are shown in Fig. 2.

The introduction of the circular hole was carried out in stages, starting from a small hole, progressively increasing the size to the required diameter and finishing the edge carefully with a hand file. The hole was stress free for all practical purposes. The hole diameter was checked for trueness with vernier calipers. In order to overcome the time-edge effect around the hole, the experiments were carried out immediately after the introduction of the hole. Optimization was also completed on the same day.



A SHELL CAST IN MOULD
B SHELL WITH PLASTER REMOVED
C SHELL MACHINED TO SIZE
D SHELL IN END FIXTURE

Fig. 2. Different stages of making shell for photoelastic experiment.

LOADING SYSTEM

The circular cylindrical shell ends were tightly gripped to outer circular end fixtures and inner split rings. The top end fixture was rigidly fixed to the top of a substantial frame while the bottom end fixture was supported below on ball bearings located on a disc in order to remove the axial load on the shell. The shell was subjected to a pure torque through loads applied to arms attached on either side of the lower end fixture by means of a pulley/pan and dead weight arrangement (Fig. 3).

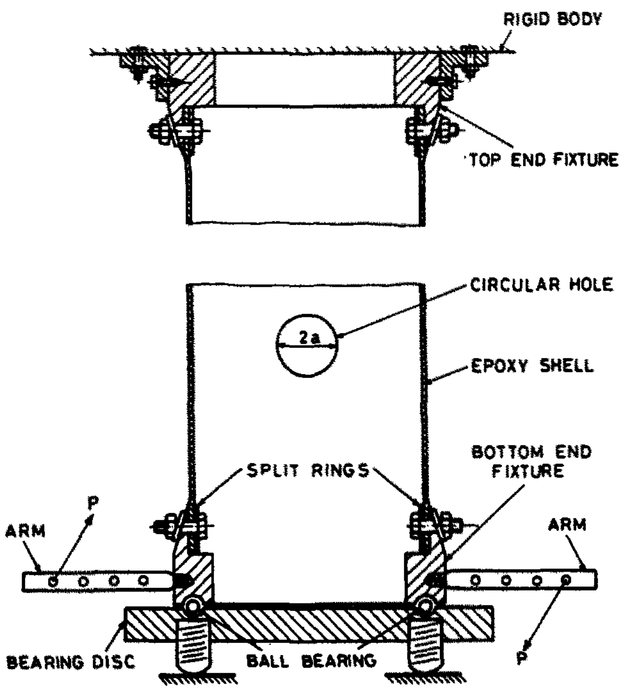


Fig. 3. Torsional loading system.

TYPE OF POLARISCOPE

Conventional transmission polariscope is clearly unsuitable for a study of the present problem. A polariscope, described in [2], with the polarizer, Quarter Wave Plate elements having their planes parallel to the plane of the hole and located at the centre of the shell with the polarization axes properly aligned with respect to the load axis, was made use of in the present study. A diffused light source with a 55 W sodium vapour lamp was used on one side of the shell while the analyzer-Q.W. plate system was located on the other side of the shell with proper alignment to complete the polariscope. With the present arrangement both dark and bright field isochromatic patterns could be observed. The fringes were recorded using a bellow-type camera using a lens with a focal length of 625 mm.

CONSTRAINTS FOR OPTIMIZATION

As in any optimization process, the design constraints for optimization are stipulated for the given problem. A constraint is imposed that the optimized hole should be tangent to the original circular hole at the points where the membrane stress is a maximum on the boundary of the circular hole and can extend in other directions.

OPTIMIZATION PROCESS

The optimization process starts with the viewing of the isochromatics around the circular hole in the shell under torsional loading using the polariscope described above. From a study of the dark and bright field isochromatics, the low and high stress regions around the hole boundary on the tensile and compressive segments are established. Keeping in view the geometric constraints stipulated for the problem, one starts filing away material from the points where the membrane stress is at a minimum, say, on the tensile segment of the boundary with the hole region in view through the analyser. This operation redistributes the stresses bringing down the maximum stress level along that segment. The operation is continued until an isochromatic fringe coincided with the tensile boundary. The same operation is continued on the other segments, maintaining symmetry at all times.

EVALUATION

The degree of optimization is evaluated quantitatively by a coefficient of efficiency K_{eff} defined as [4–6]

$$K_{\text{eff}} = \int_{S_0}^{S_1} \sigma_\gamma^{\pm} \, ds/\sigma_{\text{all}}(S_1 - S_0)$$

where σ_γ is the tangential stress on the side edge, σ_{all} represents the maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively), S_0 and S_1 are the limiting points of the segment of the boundary.

EXPERIMENTAL PROCEDURE

Two epoxy shells of 330 mm height were cast as described earlier for the two-dimensional photoelastic experiments with shell A having thickness $t = 2.4$ mm and mean radius $R = 82.45$ mm and shell B having $t = 2.5$ mm and $R = 82.5$ mm. Five circular and corresponding square holes were considered. For shell A, two circular holes of diameter $2a$ and corresponding square holes of side length $2a$ with $2a = 36.5$ mm and 51 mm resulting in β values of 0.83 and 1.16 were chosen for the study, while for shell B, the corresponding values were $2a = 22.5$ mm, 30 mm and 45 mm with β

values of 0.5, 0.66 and 1.0. The material fringe constants f_σ of the shells A and B were measured to be 11.8 and 13.4 kgf/cm-fr., respectively. Two holes of same diameter were introduced on diametrically opposite sides on the shell equidistant from the shell ends. Optimization of the hole shape was carried out starting from a circular hole of smallest diameter and progressively increasing to larger diameter holes after each optimization. Photographic recording of the dark and bright field isochromatic fringes was made before and after optimization.

RESULTS AND CONCLUSIONS

Typical bright field isochromatic pattern for circular, square and the corresponding optimized holes in a circular cylindrical shell under torsion is shown in Fig. 4. The stress distributions around circular, square and optimized holes for various a/R values are given in Fig. 5. The membrane s.c.f. values for circular, square and optimized holes together with the available analytical and experimental results for various values of β are presented in Fig. 6. The empirically developed optimized hole geometries have been fitted with a combination of circles of different diameters and common tangents at the points of intersection. Such geometries of optimized shapes are shown in Fig. 7 for different values of β.

The isochromatic fringe pattern for $\beta = 1.16$ in Fig. 4 shows the locations of peak stresses for circular and square holes to be at $\gamma = 61°$ and 82° respectively. For the optimized hole, an isochromatic fringe is found to follow the hole edge very closely. The membrane stress distributions presented in Fig. 5 for various values of β show that, as β increases, the peak stress value increases with its location progressively moving towards $\gamma = 90°$ line for circular and square holes. The optimized holes show a significantly lower s.c.f. and a uniform stress distribution on major portion of the hole boundary.

Comparison of the present circular hole values with the corresponding analytical results of Van Dyke [7] quoted by Peterson [1] shows good correlation for the range

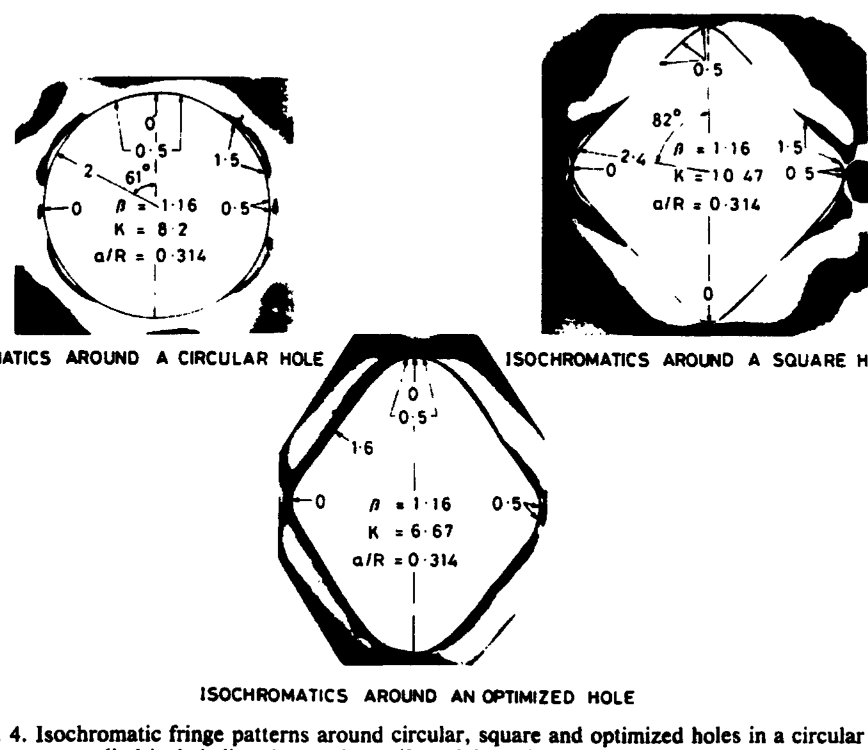


Fig. 4. Isochromatic fringe patterns around circular, square and optimized holes in a circular cylindrical shell under torsion ($a/R = 0.314$; $t/R = 0.029$; $\beta = 1.16$).

of β values considered, even though the present values are outside the range of validity of Van Dyke's solution (Fig. 6). However, surprisingly, Van Dyke's [7] as well as Savin's values [7] at $\theta = 45°$ are falling above the membrane s.c.f. values of all others including those of Peterson [1]. The three dimensional photoelastic results of Jessop et al. [8] are found to be underestimates for $\beta \leq 0.65$ but show a large over estimate for $\beta > 0.65$. The results of Houghton et al. [9] by the frozen stress technique bring out very close correlation. Aleksandrov et al., value [10] for $\beta = 0.64$, obtained from a photoelastic coating experiment on duraluminium and steel tubes, is in reasonable agreement with the present results. The preliminary values reported earlier [3] (and presented in a corrected form in Fig. 6) show a trend similar to the one observed here.

Comparison of the s.c.f. values for circular and optimized holes shows that the optimization process has brought down the s.c.f. significantly for all values of β. The amount of reduction is found to be almost constant for all hole sizes studied here. With the geometric constraints specified above, the optimized shape for $\beta = 0.5$ is found to be a double barrel hole with rounded corners with axial and diagonal symmetry and for $\beta = 0.66$, 0.83, 1.0 and 1.16, it is a rhombus with rounded corners (Fig. 7). From Fig. 7, it is seen that a near-square hole with rounded corners itself is very close to the optimum shape for $\beta < 0.4$. For square holes, Savin's results [11], which are valid for small values of β, are in close agreement with the present experimental values.

As β changes from 0.36–1.16, it is observed that (a) the optimum shape which initially

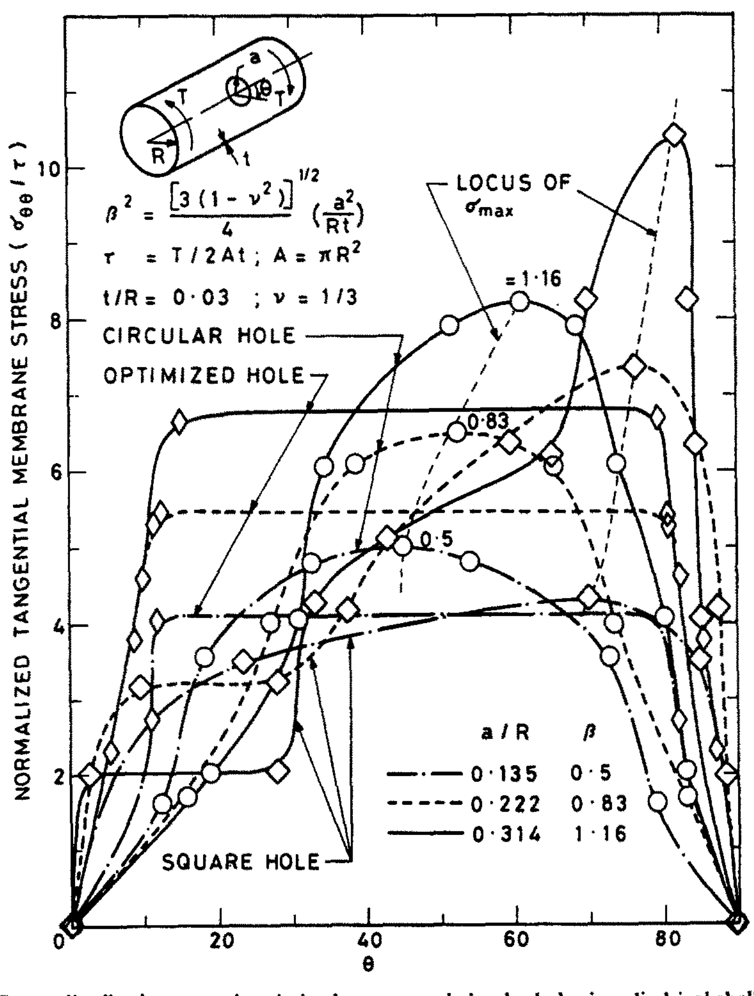


Fig. 5. Stress distribution around optimized, square and circular holes in cylindrical shell under torsion.

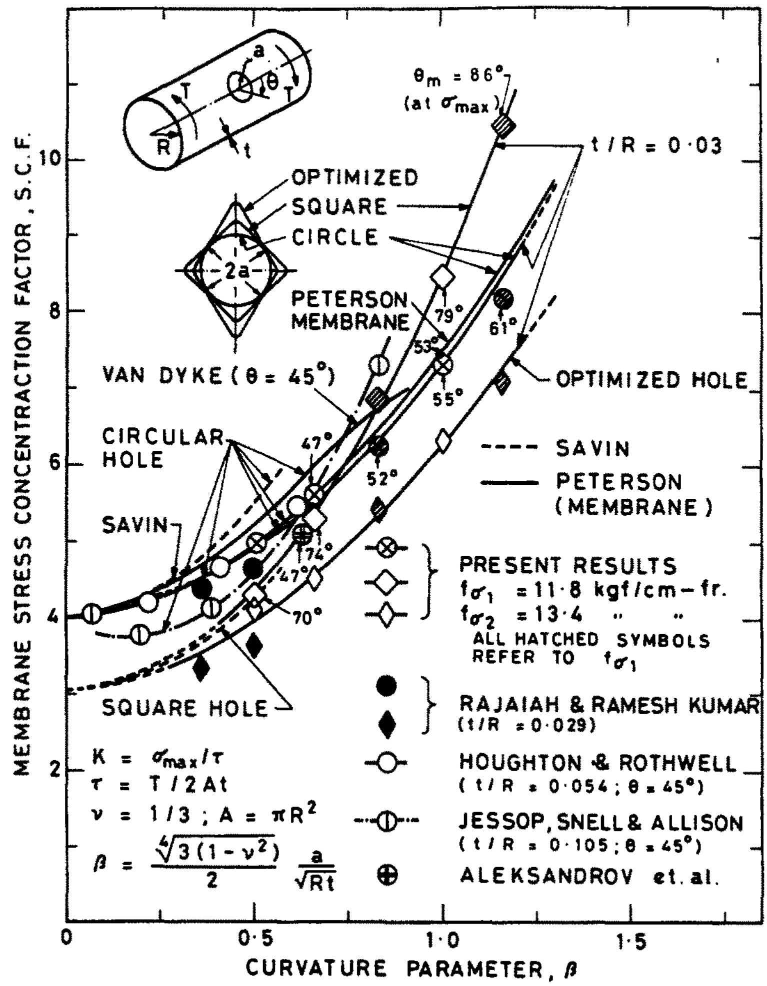


Fig. 6. Comparison of results.

assumes the form of a near-square becomes a double barrel and then finally a rhombus with rounded corners (Fig. 7); however, in all the cases, the maximum hole width in the hoop direction attains a constant value of 2.28*a*!, (b) the percentage reduction in s.c.f. with respect to the circular hole varies from 23%–18% and with respect to square hole it is 6%–33%, (c) the percentage increase in hole area of the optimum shape changes from 9%–41%, and (d) the coefficient of efficiency varies from 0.8–0.91 (Fig. 8) for optimized holes.

As mentioned earlier, the hole shape optimization has been carried out considering only the effect of the predominantly large membrane stresses. A recent study [12] incorporating the bending stress in the process of optimization has shown that the bending stress at peak membrane stress location also decreases during optimization with the decrease being a maximum at $\beta = 0.5$ and becoming negligible beyond $\beta = 1$. Thus one may conclude that the total (membrane plus bending) s.c.f. also decreases during the process of optimization in the range of β values investigated.

The present photoelastic investigation of shells with cut outs leads to an over estimation of fringe order on the hole edges by about 2–4 per cent due to thickness variation on the hole edges arising from the way the hole is introduced in the shell, with the edge surface always oriented in the direction of the optical axis. The results presented here have been corrected for this error.

One may often feel that a square hole with one of its diagonals along the axis of the

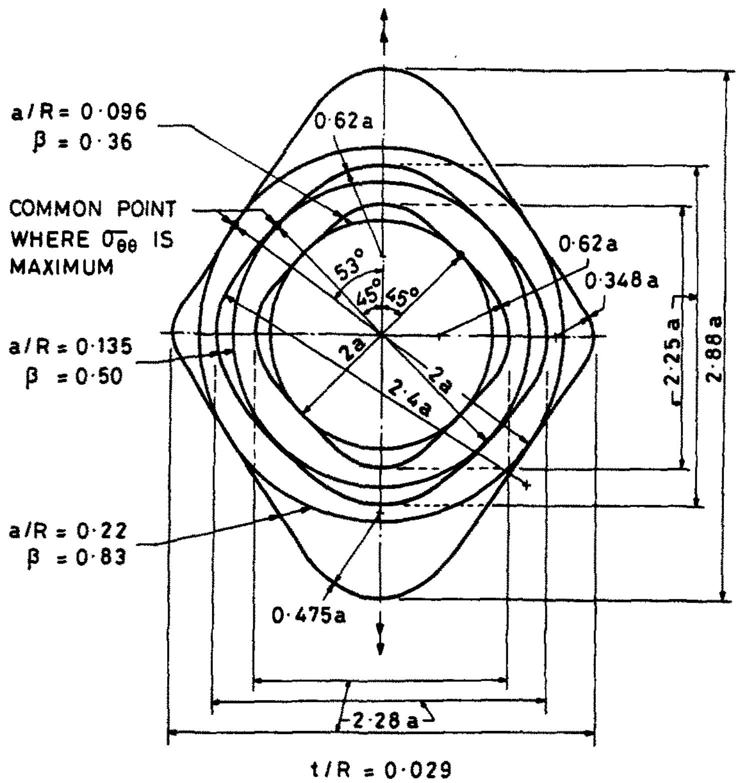


Fig. 7. Optimized hole geometries.

shell would give rise to a lower s.c.f. than the corresponding circular hole in circular cylindrical shells under torsion. Our study shows that it is true only for $\beta < 0.75$. For $\beta > 0.75$, the s.c.f. for square hole becomes much larger than that for the corresponding circular hole. The present optimized hole shapes, however, lead to consistently lower s.c.f. values.

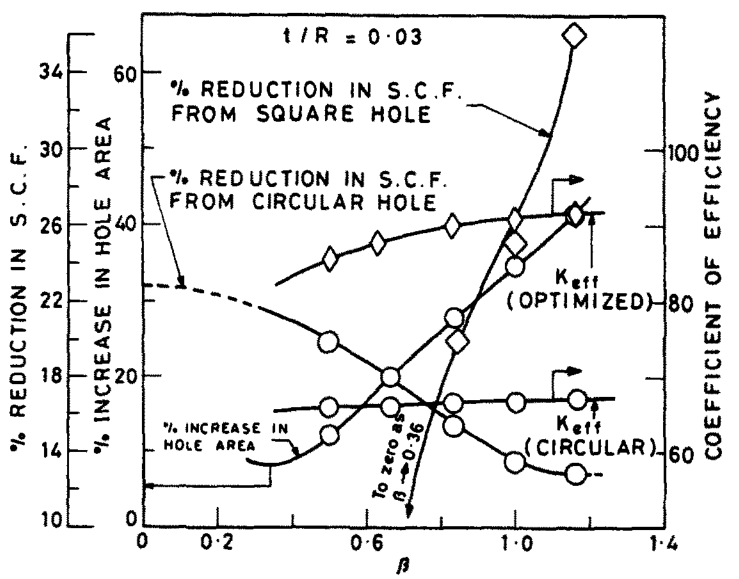


Fig. 8. Percentage reduction in s.c.f., percentage increase in hole area and coefficient of efficiency for optimized holes in circular cylindrical shell under torsion.

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